MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

TMT 1181 – MATHEMATICAL TECHNIQUES II

(All sections / Groups)

4TH MARCH 2016
3.00 p.m. – 5.00 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **6 pages** only including the cover page with **4 Questions** and an **Appendix**.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **START EACH QUESTION ON A NEW PAGE.**

4. Show all steps.

Question 1

a. A ***differential equation*** (DE) is said to be ***linear*** if it can be written in the form of

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \qquad \text{Equation (1)}$$

By referring to Equation (1), determine the ***linearity*** of the following DE:

i. $yy''-y=0$ **[1 mark]**

ii. $\frac{d^3 p}{dt^3}+\frac{dp}{dt}+p^2=0$ **[1 mark]**

b. Give one example for differential equation which is considered as ***ordinary differential equation***.

[1 mark]

c. Consider the following ***initial value problem***:

$$M(x,y)dx + N(x,y)dy = 0; \qquad y(5)=1$$

where $M(x,y)=2xy$ and $N(x,y)=x^2$.

i. Show that the above ***differential equation*** is ***exact***. **[2.5 marks]**

ii. Let $F(x,y)=\int M(x,y)dx + g(y)$. Find $g(y)$ and $F(x,y)$. **[2.5 marks]**

iii. Find the value c from $F(x,y)$ obtained in (ii). **[2 marks]**

Continued...

Question 2

a. Show that the ***partial fraction*** of $\dfrac{\frac{1}{s}+6s}{s^2+1}$ is $\dfrac{1}{s}+\dfrac{5s}{s^2+1}$. **[2.5 marks]**

b. From result obtained in part (a), solve the following ***initial value problem*** using ***Laplace transform.***

$$y''(t)+y(t)=1,\quad y(0)=6,\quad y'(0)=0$$ **[2.5 marks]**

c. Find the ***Fourier Series*** representation for the following periodic function of period $(-L,L)$.

$$f(x)=\begin{cases}0, & -\pi \le x<0\\ \pi, & 0\le x<\pi\end{cases}$$ **[5 marks]**

HINT:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi}{L}\right)x+b_n\sin\left(\frac{n\pi}{L}\right)x\right),$$

$$a_0=\frac{1}{L}\int_{-L}^{L}f(x)dx\ ;\ a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\left(\frac{n\pi x}{L}\right)dx\ ;\ b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$\sin n\pi = 0$ for $n=1,2,...$; $\cos n\pi=(-1)^n$ for $n=1,2,...$

Continued...

Question 3

Consider the following table where $f(x)=\frac{1}{x}$.

x	2	2.5	4
$f(x)$	A	B	C

[**NOTE:** Show the results of your calculations correct to **FOUR** decimal places.]

a. Find the values of A, B and C in the above table. **[0.5 mark]**

b. Find the ***Lagrange*** coefficients $L_0(x)$, $L_1(x)$ and $L_2(x)$. **[3 marks]**

c. Using results obtained in part (b), find the ***second interpolating polynomial*** $P_2(x)$ by using ***Lagrange interpolating polynomial.*** **[2.5 marks]**

d. Approximate $f(1.5)$. **[1.5 marks]**

e. Hence, find the absolute error and relative error when $x=1.5$.

[2.5 marks]

Continued...

Question 4

a. Find the root of the following equation using *Newton-Raphson method* with starting point $p_0 = 3$ and tolerance $1*10^{-5}$.

$$f(x) = x^2 - 10$$

[**NOTE:** Correct your answers to **SIX** decimal places.]

[3.5 marks]

b. Consider the system of equations

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= 3 \\ 3x_1 + x_2 - x_3 &= 7 \\ x_1 - 3x_2 + 2x_3 &= 0 \end{aligned}$$

i. Form the ***augmented matrix*** of the above system. **[1 mark]**

ii. Hence, solve the above system by using ***Gauss-Jordan*** elimination method. **[5.5 marks]**

Continued...

APPENDIX

TABLE A: LAPLACE TRANSFORMS OF SOME ELEMENTARY FUNCTIONS

Function, $f(t)$	Laplace transform, $F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{\alpha t}$	$\frac{1}{s-\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$f'(t)$	$s \cdot F(s) - f(0)$
$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$
$t^n \cdot f(t)$	$(-1)^n \cdot \frac{d^n F(s)}{ds^n}$

FORMULAS:

Newton-Raphson Method

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

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